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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number one (Due: Sat. at 1pm October 6)

Ayman Badawi

QUESTION 1. Let G be a group.

(i) Suppose that $a \in G$ such that $a^n = e$ for some $n \in Z^+$. Let $m = |a|$. Prove that $m \mid n$.

(ii) Let $a \in G$ and suppose that $a^m = a^n$ for some integers m, n where $m \neq n$. Prove that $|a| < \infty$

(iii) Let $a, b \in G$ such that $ab = ba$. Let $m = |a|$ and $n = |b|$ where $gcd(n, m) = 1$. Prove that $|ab| = nm$

(iv) Give me an Example of a finite group G where $a, b \in G$, $gcd(|a|, |b|) = 1$ but $|ab| \neq |a||b|$ (we should conclude that $ab = ba$ in (ii) is crucial!!)

(v) Let $a \in G$. Prove $|a| = |a^{-1}|$. (First assume $|a| = \infty$ and prove $|a^{-1}| = \infty$. Then assume $|a| = m < \infty$ and prove $|a^{-1}| = m < \infty$)

(vi) Assume G is an infinite cyclic and $G =< a >$ for some $a \in G$. Let $b \in G$ such that $b \neq e$. Prove $|b| = \infty$.

(vii) Give me an example of an infinite abelian group say G such that G has exactly two elements, say a, b, where $|a| = |b| = 3$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`